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First version: Apr 09, 2013 at 21:30 by **Bryan D'Amico**
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Collection

Standard tableaux

Description

The inversion number of a standard tableau as defined by Haglund and Stevens. Their inversion number is the total number of inversion pairs for the tableau. An inversion pair is defined as a pair of cells (a,b) , (x,y) such that the content of (x,y) is greater than the content of (a,b) and (x,y) is north of the inversion path of (a,b) , where the inversion path is defined in detail in [1].

References

[1] **Haglund, J., Stevens, L.** *An extension of the Foata map to standard Young tableaux* www.ams.org/mathscinet/search/publdoc.html?pg1=MR&s1;=2264944

Code

```

def statistic(T):
    """
    sage: T = StandardTableau([[1,4],[2,5],[3]])
    sage: statistic(T)
    [(2, 1), (5, 4), (3, 1), (3, 2)]
    """
def inversion_path(T, i, j):
    """Return the inversion path of the cell i,
    j, as a list of column
    indices.

    Given this list 'p', cells below the path
    are those in
    'T[k][l]' with '0<=k<len(p)' and 'l >=
    p[k]'.

    EXAMPLES::

    sage: T = StandardTableau
    ([[1,2,4,8],[3,7,11,12],[5,9,13,14],[6,10,15,16]])

    sage: inversion_path(T, 3, 3)
    [0, 2, 2]

    sage: T = StandardTableau
    ([[1,2],[3,4]])
    sage: inversion_path(T, 1, 1)
    [0]

    sage: T = StandardTableau
    ([[1,3],[2,4]])
    sage: inversion_path(T, 1, 1)
    [1]

    """
    columns = []
    while True:
        if j == 0:
            columns.extend([0]*i)
            return columns[::-1]
        elif i == 0:
            return columns[::-1]

        elif T[i-1][j] > T[i][j-1]:
            columns.append(j)
            i -= 1

        elif T[i-1][j] < T[i][j-1]:
            j -= 1

    I = []
    for i,j in T.cells():
        c = T[i][j]
        p = inversion_path(T, i, j)
        for row, col in enumerate(p):
            c, i = T[row][col]

```

